Example problems Fatigue

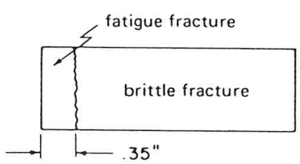
1. You are hired as a consultant to analyze a failure of a support bracket for a piece of machinery, depicted below, that was made from a high strength steel with the following properties: tensile strength, σ_T = 190 ksi, yield strength, σ_Y = 160 ksi, fatigue limit = 90 ksi, and fracture toughness, K_{IC} = 85 ksi$\sqrt{\text{in}}$. This bracket is subjected to variable tension-release loading of unknown, but constant amplitude (and sine wave shape). Examination of the fracture surface shows 0.35 in. of fatigue crack growth, followed by catastrophic failure.

(a) Name a characteristic feature of a fatigue fracture surface that may have helped you assess the fatigue versus fracture portions of crack growth.

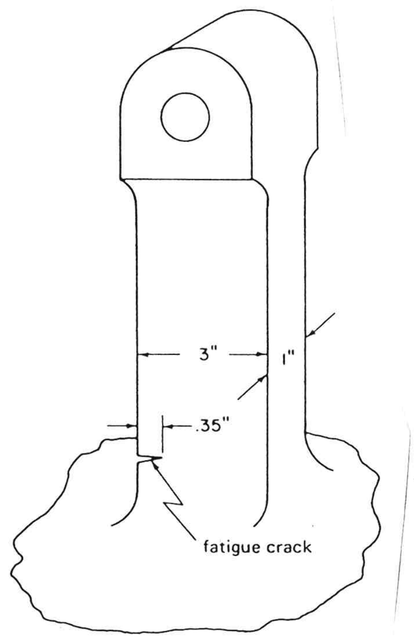
(b) From the critical crack length, determine the magnitude of the applied stress amplitude.

(c) Construct an approximate S-N curve from the data above and estimate the number of cycles that occurred prior to catastrophic failure.

(d) How must the stress amplitude be reduced in order to eliminate the fatigue problem. What assumption are you making by using an S-N curve to answer this question? What is a more conservative approach?



Fracture surface



Support bracket

2. Answer the following questions on small fatigue cracks and toughening mechanisms.

(a) Why is the fatigue behavior of small cracks important to consider in a damage tolerant approach to fatigue life prediction?
(b) List intrinsic and extrinsic toughening mechanisms. Explain how these mechanisms may be affected by crack length.
(c) Which of these mechanisms improve fracture toughness, but not fatigue resistance? Explain why these particular mechanisms are not effective during fatigue crack growth.
(d) For sine wave fatigue loading, draw two cycles of stress intensity as a function of time for an unshielded crack and for a crack of the same length under the same applied loads with oxide induced closure. Mark the maxima, minima, cycle lengths, and closure points of the cycles.

3. When a fast breeder nuclear reactor is shut down quickly, the temperature on the surface of a number of AISI 304 stainless steel components drops from 600°C to 400°C in less than a second, while the bulk of these components takes several seconds to cool. The Coffin-Manson low-cycle fatigue life of the steel is described by:

$$\sqrt{N_f}\,\Delta\varepsilon_{pl} = 0.2$$

where N_f is the number of cycles to failure and $\Delta\varepsilon_{pl}$ is the plastic strain range. Estimate the number of fast shutdowns that the reactor can sustain before serious cracking or failure will occur. You may assume a thermal expansion coefficient for steel, $\alpha = 1.2\text{x}10^{-5}$/K and a yield strain of $0.4\text{x}10^{-3}$ at 400°C.

4.

Coal gasification processes require massive welded steel pressure vessels to operate safely at high temperatures and pressures in hostile environments containing H_2S, H_2, H_2O, CH_4 gases, etc. One such vessel is fabricated by forming curved sections of a quenched and tempered 2¼Cr-1Mo steel (ASTM A542 Class 3) and electroslag welding them together with longitudinal welds and capping the cylinder with hemispherical ends. The vessel is 200 ft. high, of diameter (d) 20 ft. with a constant wall thickness (t) of 8 in. During frequent start-up and shut-down, the internal gas pressure (p) in the vessel varies in cycles from zero to 2000 psi, while maintaining a vessel wall temperature of 300°C. However, as with all large scale welded structures, cracks are likely to initiate at imperfections in the weld, and propagate sub-critically along the axis of the cylinder perpendicular to the maximum stress direction (Fig. 1). Accordingly, <u>prior</u> to entering service, a complete and successful ultrasonic NDT evaluation of the vessel is performed by an operator who claims to reliably detect all flaws of surface length greater than 0.02 in. long.

To ensure safe operation, estimates must be made of the lifetime before such cracks reach critical size and catastrophic failure occurs. Two approaches are feasible:

A) Initially inspect the weld using ultrasonics (limit of detection ~0.02 in.) and determine the number of cycles to failure (N_f), such that periodic inspection can be carried out at half the number of cycles ($N_f/2$).

B) Proof test the cylinder at a higher internal pressure (p_{proof}) every 10^5 cycles to estimate the flaw sizes (Fig. 2).

Using the data given, you are asked to:

i) Determine the critical crack size (a_c) for catastrophic failure under operating conditions;

ii) estimate the safe inspection interval (i.e., $N_f/2$) using the initial NDT procedure (A);

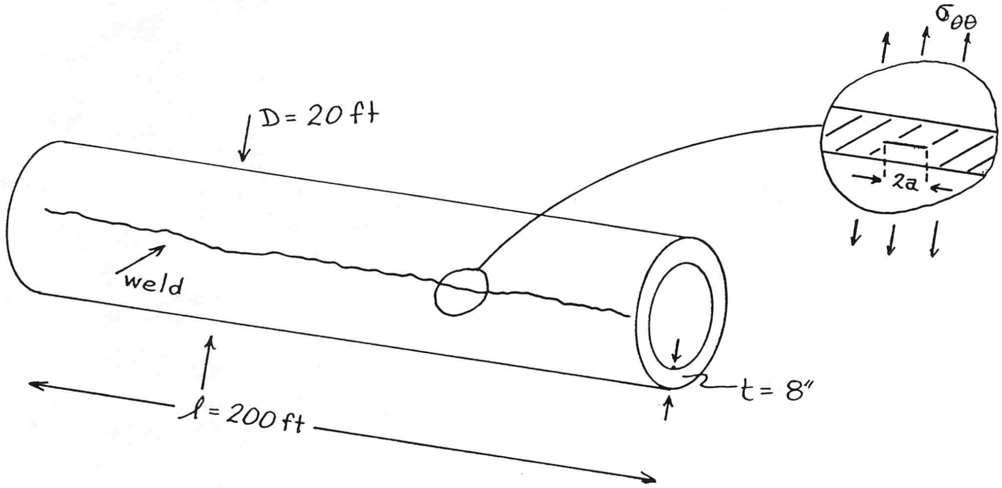
iii) determine the proof test pressure (p_{proof}) to ensure that failure does not occur within the next 10^5 cycles (Fig. 2) using NDT procedures (B).

You may assume that the proof test cycle in B does not affect subsequent crack growth behavior. **Is this latter assumption conservative or non-conservative?**

<u>Mechanical Property Data for ASTM A542 Class 3</u>

Elastic modulus	E	= 30 x 10^3	ksi	(ambient temperature)
		= 28 x 10^3	ksi	(300°C)
Yield strength	σ_y	= 65	ksi	(ambient temperature)
		= 60	ksi	(300°C)
U.T.S.	σ_u	= 80	ksi	(ambient temperature)
Fracture toughness	K_{Ic}	= 50	ksi√in	(ambient temperature)
		= 60	ksi√in	(300°C)

Fatigue crack propagation rates at $R = 0$ in environments of gaseous hydrogen and 5% hydrogen sulfide at 300°C given by $da/dN = C\Delta K^m$, where $m = 3$, $C = 10^{-10}$ for units of ksi√in for ΔK and inches/cycle for da/dN.



$$K_I = \sigma^{\infty} \sqrt{\pi a}$$

Fig. 3.1

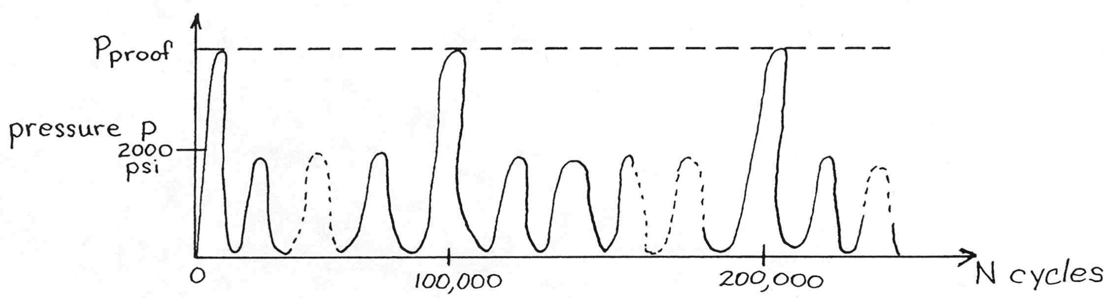


Fig. 3.2

Solutions to example problems

1) (a) Striations are characteristic of fatigue fracture (periodic ridges $\perp$ to crack growth direction.

(b)

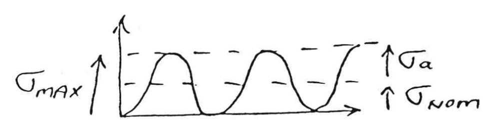
$$K_{IC} = \sigma_\infty \sqrt{C}\,(1.1) \qquad \sigma_\infty = \frac{85{,}000 \text{ psi in}^{1/2}}{(1.1)(.35 \text{ in})^{1/2}}$$

$$\boxed{\sigma_\infty = 131{,}000 \text{ psi} = \sigma_{MAX}}$$

Even though $R_C \cong \frac{K_{IC}^2}{(T.S.)^2} \cong 0.20 \text{ in.}$

[This indicates a relatively large plastic zone compared to C. We will assume the analysis to be correct since R_C is not excessively large.]

Since the loading is tension release:



$$\boxed{\sigma_A = \sigma_{Nom} = \tfrac{1}{2}\,\sigma_{MAX} = 65{,}500 \text{psi}}$$

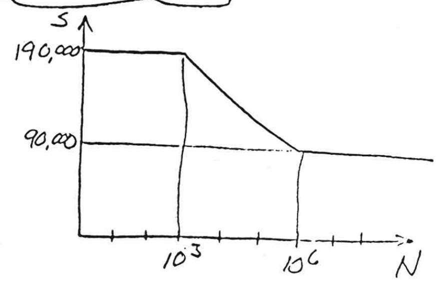
C

Using the Goodman-relation to judge the effective tension-compression amplitude, σ_{ao}

$$\frac{\sigma_a}{\sigma_{ao}} + \frac{\sigma_m}{T.S.} = 1 \qquad \frac{65,500}{\sigma_{ao}} + \frac{65,500}{190,000} = 1$$

$$\boxed{\sigma_{ao} = 100,000 \text{ psi}}$$

Approximate the S-N curve by T.S. at 10^3 cycles and σ_ℓ at 10^6 cycles.



$$\sigma_{ao}^{m} N = C \qquad m \log \sigma_{ao} + \log N = \log C$$

$$m = -\frac{\Delta \log N}{\Delta \log \sigma_{ao}} = -\frac{3}{-.324}$$

$$\boxed{m = 9.25}$$

$$\sigma_{ao}^{9.25} N_f = (90,000)^{9.25} \cdot 10^6$$

$$N_f = \left(\frac{90,000}{100,000}\right)^{9.25} 10^6 \qquad \boxed{N_f = 372,000 \text{ CYCLES}}$$

(d)

[Since we used the Goodman Relation to estimate σ_{ao}, this value of N_f underestimates the actual lifetime].

We want to limit the maximum load so that failure will not occur: $\sigma_{ao} \leq \sigma_e$

$$\sigma_a = \sigma_m - \tfrac{1}{2}\sigma_{max} \text{ (for tension-release).}$$

$$\frac{\sigma_a}{90{,}000} + \frac{\sigma_a}{190{,}000} = 1 \qquad \sigma_a = 90{,}000\left(\tfrac{19}{28}\right)$$

$$\boxed{\begin{array}{l} \sigma_a^{\text{LIMIT}} \leq 61{,}000\,\text{psi} \\ \sigma_{MAX} \leq 122{,}000\,\text{psi} \end{array}}$$

[A conservative estimate since the Goodman Relation was used.]

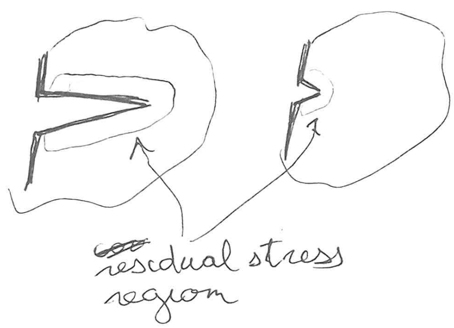
An S-N approach assumes that no cracks are initially present. A damage tolerant approach, on the other hand, assumes a maximum crack length, which is a more conservative assumption.

2) (a) Much of the fatigue life may be spent when the crack is "small", particularly crack growth that occurs prior to detection by a NDT. If these cracks propagate below ΔK_{TH} for long cracks, this must be included in a conservative analysis.

(b)

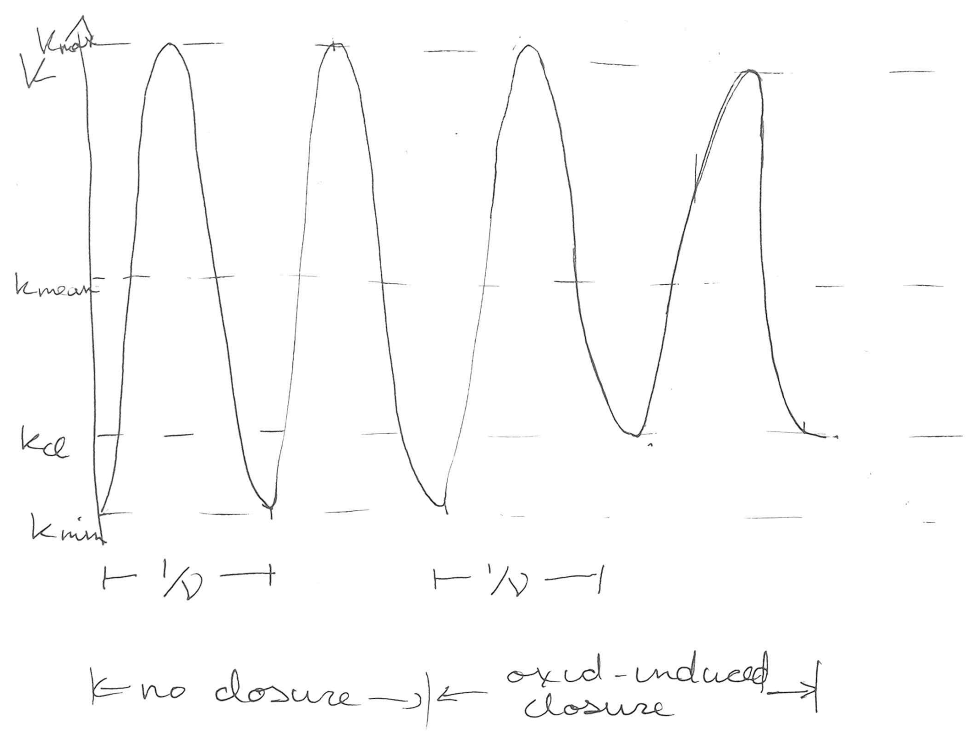
intrinsic	extrinsic
plasticity	fiber bridging
void formation	grain bridging
phase transformation	oxide wedging
microcracking	crack deflection (roughness)

Many of these mechanisms scale with crack length up to a point because for instance the number of bridges behind the crack or the length of a compression region created by microcracking, voids, phase trans. or plasticity increases with crack length.



Fatigue cycling dimanishes many of these mechanisims – fiber bridging, grain ~~br~~ bridging & crack deflection. The cycling causes breaking of fibers & wear of the pull out grains & fibers. After wear, there isn't any frictional resistance. This wear by sliding also affects resistance due to crack deflection, although the deflection also increases the length of the crack path, which isn't affected by fatigue

(d)



3) $\Delta\epsilon_{Tot} = \Delta\epsilon_{pl} + \Delta\epsilon_{el}$

$\Delta\epsilon_{pl} = \alpha\Delta T - \Delta\epsilon_{yield}$

$= 1.2\times10^{5}/K\,(600-400^\circ C) - 0.4\times10^{3}$

$= 2\times10^{-3}$

$N_F^{1/2}\,\Delta\epsilon_{pl} = 0.2$

$N_F = \left(\frac{0.2}{\Delta\epsilon_{pl}}\right)^{-1/2}$

$= \left(\frac{0.2}{2\times10^{-3}}\right)^{-1/2}$

$= 10^4$ cycles

4.

— ~~Hoop~~ stress $\sigma_{\theta\theta} = \frac{pr}{t} = \frac{2000 \times 120''}{8''} = 30 \text{ ksi}$

as $K_I = \sigma_{\theta\theta}\sqrt{\pi a}$,

at $K_I = K_{Ic}$, $a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_{\theta\theta}}\right)^2 = \frac{1}{\pi}\left(\frac{60}{30}\right)^2 = 1.27$

i) $\boxed{\therefore \text{ critical crack size } 2a_c = 2.54''}$

— For fatigue lifetime,

use $da/dN = C\,\Delta K^m$

$\therefore \frac{da}{dN} = C\left(\Delta\sigma\sqrt{\pi a}\right)^m$

$$\int_{a_0}^{a_c} \frac{da}{a^{m/2}} = C\left(\Delta\sigma\sqrt{\pi}\right)^m \int_0^{N_f} dN$$

$$\therefore N_f = \frac{1}{C\,\Delta\sigma^m\,\pi^{m/2}}\left[\frac{1}{a^{\frac{m}{2}-1}} - \frac{1}{a^{\frac{m}{2}-1}}\right]_{a_0}^{a_c}$$

$$\therefore N_f = \frac{2}{(m-2)\, C\, \Delta\sigma^m\, \pi^{m/2}} \left[\frac{1}{a_o^{\frac{m-2}{2}}} - \frac{1}{a_f^{\frac{m-2}{2}}} \right]$$

said $m = 3$, $C = 10^{-10}$, $\Delta\sigma = \sigma_{max} = 30\,ksi$

$a_f = a_c = 1.27''$

$$N_f = \frac{2}{(1)(10^{-10})(30)^3\, \pi^{1.5}} \left[\frac{1}{a_o^{1/2}} - \frac{1}{a_f^{1/2}} \right]$$

$$= \frac{2 \times 10^{10}}{2.7 \times 10^4 \times 5.57} \left[\frac{1}{\sqrt{a_o}} - \frac{1}{\sqrt{1.27''}} \right]$$

ii) assume a_o = largest undetected defect $= 0.02''/2$

$= 0.01''$

$$\therefore N_f = 1.33 \times 10^5 \left[10 - 0.89 \right]$$

$$= 1.21 \times 10^6 \text{ cycles}$$

note how little effect the value of a_f has on N_f, compared to a_o

$\therefore$ iii) inspection period $= N_f/2 = 6 \times 10^5$ cycles

iii) To insure that failure does not occur within a given 10^5 cycle, we must determine the initial crack size a_o' such that this crack does not reach $a_f = a_c$ within 10^5 cycles.

$$\therefore \text{ using } \quad N_f = 1.33\times10^5 \left[\frac{1}{\sqrt{a_o}} - \frac{1}{\sqrt{a_f}}\right]$$

$$\therefore 10^5 = 1.33\times10^5 \left[\frac{1}{\sqrt{a_o'}} - \frac{1}{\sqrt{1.27}}\right]$$

$$\therefore a_o' = 0.37''$$

for failure during proof test, where $K_{Ic} = \sigma_{proof}\sqrt{\pi a_o'}$

$$\therefore \sigma_{proof} = \frac{K_{Ic}}{\sqrt{\pi a_o'}} = \frac{60}{\sqrt{\pi\, 0.37}} = 55.7 \text{ ksi}$$

∴ if initial cracks are $> a_o'$, part will fail in proof test

" " " $< a_o'$, part will fail at $N_f > 10^5$ cy

iii) ∴ proof pressure $P_{proof} = \sigma_{proof} \frac{t}{r} \geq 3710$ psi

Likely to be conservative as overload due to proof test will enlarge plastic zone & may cause subsequent crack growth retardation